EXERCISE SHEET 3

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1. Let H_d be the degenerate affine Hecke algebra, and H_d^{aff} be the affine Hecke algebra. Show that
$$Z(H_d) = \mathbb{C}[X_1, \dots, X_d],$$
and
$$Z(H_d^{\mathrm{aff}}) = C[X_1^{\pm}, \dots, X_d^{\pm}]^{S_d}.$$
2. Verify that H_d acts on $M \otimes V^{\otimes d}$. Show that
$$\Omega = \frac{1}{2}(\Delta(C) - C \otimes 1 - 1 \otimes C)$$
on $M \otimes V$, where C is the Casimir element in $U(\mathfrak{g})$. Use this to find a finite-dimensional quotient of H_d acting on $M^{\mathfrak{p}}(\lambda) \otimes V$ for $\mathfrak{p} \subset \mathfrak{gl}_n$ a parabolic, and λ any integral weight of your choice.
3. Guess some relations in $\mathbb{W}_d$ from the action.
4. Consider $\partial_i \colon \mathbb{C}[X_1 \dots, X_d] \to \mathbb{C}[X_1, \dots, X_d]$ for $1 \leq i \leq d-1$, defined by
$$\partial_i(f) = \frac{f - s_i(f)}{X_i - X_{i+1}}.$$
Show that $\partial_i^2 = 0$, and that the operators ∂_i satisfy the braid relations. Compute the subalgebra of $\mathrm{End}_{\mathbb{C}}(\mathbb{C}[X_1, \dots, X_d])$ generated by the operators ∂_i and multiplication with X_is. Compare with $R_{\underline{d}}$ with $e > 3$, and $\underline{d} = (0, \dots, 0, k, 0, \dots)$ for any k.
5. Show that
$$A_{\underline{d}} \cong \bigoplus_{(\widehat{\lambda}, \widehat{\mu})} \mathrm{Ext}^*(p_* \underline{\mathbb{C}}_{Q(\widehat{\lambda})}, p_* \underline{\mathbb{C}}_{Q(\widehat{\mu})}),$$
where $p = p_{\widehat{\lambda}}$ is the forgetful functor $Q(\widehat{\lambda}) \to \mathrm{Rep}_{\underline{d}}$.
6. Construct a representation of H_d on $\mathbb{C}[X_1, \dots, X_d]$. Is it faithful?